



## Math Objectives

- Students will interpret a confidence level as the degree of confidence in the process used to produce an interval that will contain the true mean of the population.
- Students will recognize that as the confidence level increases, on average the confidence interval increases in width.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practices).

## Vocabulary

- confidence interval
- confidence level
- population parameter

## About the Lesson

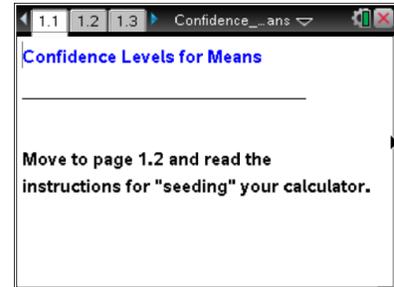
- This lesson involves generating a confidence interval for the mean of a population from a random sample of size  $n$  and considering how certain one can be that this interval would contain the actual population mean.
- As a result, students will:
  - Experiment with different confidence levels and finally investigate the relationship between confidence levels and sample size.
  - Use the actual population mean to check their estimates and to develop an understanding of the meaning of confidence level.

## TI-Nspire™ Navigator™ System

- Send the .tns file to students.
- Use Screen Capture to display multiple sets of confidence intervals.
- Use Quick Poll to check student understanding and prior knowledge.

## Prerequisites:

- Calculation of a confidence interval
- Sampling distribution of a sample statistic.



## TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

## Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- In the *Graphs & Geometry* application, you can hide the function entry line by pressing  .

## Lesson Materials:

### *Student Activity*

Confidence\_Levels\_Student.pdf

Confidence\_Levels\_Student.doc

### *TI-Nspire document*

Confidence\_Levels.tns

Visit [www.mathnspired.com](http://www.mathnspired.com) for lesson updates and tech tip videos.



## Discussion Points and Possible Answers

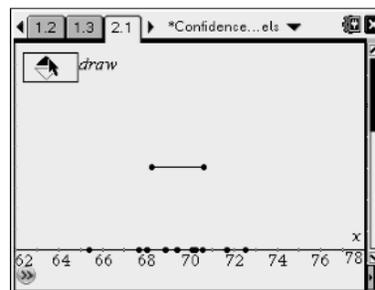
**Teacher Tip:** Confidence intervals are the results of a process that allows us to make an inference about unknown parameters for a population and are calculated from a random sample drawn from that population. You might want to use Quick Poll to have students practice generating a confidence interval.

**Tech Tip:** Page 1.2 gives instructions on how to seed the random number generator on the TI-Nspire. Page 1.3 is a *Calculator* page for the seeding process. Ensuring that students carry out this step will prevent students from generating identical data. (Syntax: RandSeed #, where # is a number unique to each student.)

### Move to page 2.1.

1. Use the arrow (▲) to draw a sample. The segment represents the calculated confidence interval for that sample.
  - a. What do the points on the horizontal axis represent?

**Answer:** The points represent the elements from a population.



**Teacher Tip:** The points might not be distinct due to the scaling; several of them might lie very close to each other and appear as one point, so the total number of elements in the sample might not appear to be 10.

- b. Use the confidence interval to estimate the population mean. How confident are you that your estimate really captures the true mean of the population?

**Sample Answer:** The population mean could be anywhere in the confidence interval, for example, from about 68 to 71. Students might give the center of the interval as their answer, i.e., 69.5. Students might say they are very confident because they believe the confidence interval does capture the population mean, or they might be unsure because they only looked at one sample.

**Teacher Tip:** Students' estimates should reflect the span of the interval, not just the center; this might not be clear at this point but should become very clear by the end of the activity. If students do give only the center of the interval, pursue the "how confident" question. Comparing the confidence of a student who guessed only one point to that of someone with an interval should help students see the value of interval estimates. The purpose of the question is to get them started thinking about how confident they are that the process will produce an interval that contains the population mean.



- c. Although in real situations one sample is usually all you have, suppose you could draw more than one sample from the same population so you could find more than one confidence interval for the mean of that population. Draw another sample, and estimate the population mean from this new confidence interval. Repeat this process four more times, and write down your estimate for the sample mean each time.

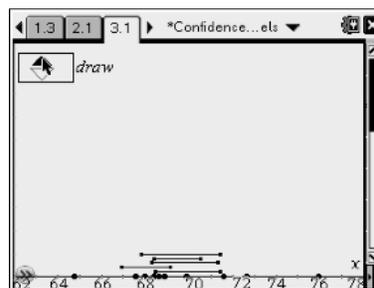
**Sample Answer:** Using the center of the confidence interval, students might respond: 70.25, 71.6, 70, 70.75, and 70.9; using the length of the interval, they might respond: 69 to 71.5; 70 to 73.2; 68.5 to 71.5; 69.5 to 72; and 69 to 72.8.

**Teacher Tip:** If the down arrow is clicked, a new random sample will also be generated; the down click does not “undo” and return to the previous sample.

**TI-Nspire Navigator Opportunity: Screen Capture**  
See Note 1 at the end of the lesson.

Move to page 3.1.

2. a. Use the up arrow (▲) to generate five confidence intervals. Using those intervals, estimate what you think the population mean is likely to be. Explain your reasoning.



**Sample Answer:** Students might give an interval estimate (69.5 to 70.5; 69.5 to 72) for any interval that overlaps among the five sample confidence intervals; they might give the center (70; 70.75) of the intervals that seems to be in common with all of the sample confidence intervals; or they might give the overlapping whole numbers in the intervals (70; 70, 71, or 72). Their reasoning might use words such as “the number 71 was in all five of the confidence intervals, so it seems likely 71 is the actual population mean.” Some might recognize that it is difficult to give one number because the lengths of the intervals keep changing (one was four units long from 69 to 73 and another was three units long from 66 to 69) and offer a small interval that seems common to the five they have generated.

**Teacher Tip:** Students should share their solutions to part a, so they can see the variability in their results and how their specific set of five confidence intervals would affect their estimates.

**TI-Nspire Navigator Opportunity: Screen Capture**  
See Note 2 at the end of the lesson.



- b. Use the up arrow ( $\blacktriangle$ ) to generate five more confidence intervals. Make another estimate of the population mean using all ten of the intervals you generated. Explain your reasoning.

**Sample Answer:** Students might again answer using the same kind of reasoning as they did in part a. They might begin to notice that 70 seems to be common to most of the intervals.

- c. Do you think all of the intervals you generated will contain the population mean? Why or why not?

**Sample Answer:** If their intervals are fairly alike, students might say yes. If one or more of the intervals is quite different from the rest (i.e., one of the ten intervals only goes from 67 to 69 and the others all seem to contain 70), they might suggest that it looks like some intervals might be so different they might not contain the population mean.

- d. Generate ten more confidence intervals, and indicate the proportion of the 20 intervals you think might be unlikely to contain the population mean. Explain your thinking.

**Sample Answer:** Students should begin to suspect that one or two of the 20 intervals do not contain the population mean because they are so far to the right or to the left of the other intervals, for a proportion of  $1/20$  or 5%, or  $2/20$  or 10%.

**Teacher Tip:** It is easier to count the intervals that seem unlikely to contain the population mean because there are fewer of them. But confidence levels are given in terms of those that are likely to contain the population mean; be sure students see the switch as they move from part d to Question 3.

**TI-Nspire Navigator Opportunity: Screen Capture**  
**See Note 3 at the end of the lesson.**

3. a. Make a conjecture about the number of intervals in a set of 50 intervals generated from the same population that you think actually will contain the population mean.

**Sample Answer:** Student conjectures will vary from “all of them seem likely” to a very small percentage.



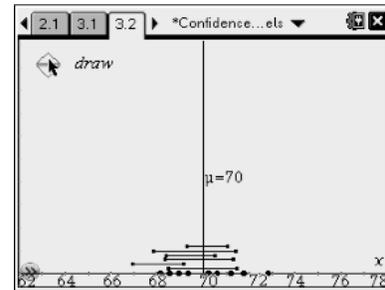
- b. Generate confidence intervals until you have 50. How do the results match your conjecture?

**Sample Answer:** Students might see that their conjectures hold, might not be able to see anything unusual, or might be able to identify that only a specific number such as two seem very different and so are unlikely to contain the true population mean, which would suggest that 96% of the confidence intervals seem likely to actually contain the population mean.

**Move to page 3.2.**

4. Page 3.2 shows a vertical line representing the true population mean, 70.

- a. What proportion of the 50 confidence intervals you had generated in Question 3 part b actually captured the true mean?



**Sample Answer:** Student answers will vary but should be near a proportion of 0.96.

- b. Make a conjecture about the number of intervals in any set of 50 intervals generated from the same population with the same sample size that you think might actually contain the population mean.

**Sample Answer:** Students might respond anywhere from none to lots, or be specific and say all but one or two, so it would be  $1/10$  (10%) or  $1/5$  (20%).

- c. Use the down arrow ( $\blacktriangledown$ ) to clear the 50 confidence intervals you generated in Question 3b. Then generate 50 new confidence intervals. How well do your results support your conjecture?

**Sample Answer:** Students might respond, for example, that four or 8% of them did not contain 70, but that they were unsure how to count the two that seemed to touch 70, raising the question of what to do about the endpoints of an interval.

**Teacher Tip:** Given the rough scale, it seems difficult to see exactly whether some intervals actually contain 70; remind students that each segment is just an approximation, and if it looks as if the “square” at the end of an interval is mostly next to 70, they might consider it as not including 70. The primary goal here is to show that for some relatively predictable proportion of time, the process of creating a confidence interval will produce intervals that do not contain the actual population mean. Do not get hung up on exact counts.

**TI-Nspire Navigator Opportunity: Screen Capture**  
**See Note 4 at the end of the lesson.**



- d. Press the down arrow (▼) to undo the confidence intervals you have on the screen. Generate 50 more intervals. Do these intervals support your conjecture? Why or why not?

**Sample Answer:** Students should find that about 45 (maybe from 43 to 48) of the intervals do contain the population mean.

5. Amie announced that in every 50 confidence intervals generated for this population, about 90% of them contain 70, the actual population mean.
- a. Do you agree or disagree with Amie? Explain why or why not.

**Sample Answer:** This should be a reasonable estimate because in Question 4, about 45 out of 50, or 90%, contained the population mean.

**Teacher Tip:** This question is designed to move students from counting the number and reporting either that number (46) or a fraction (46/50) to the language of percent.

- b. Use the down arrow (▼) to undo the confidence intervals you generated in Question 4. Generate a new set of 50 intervals and see whether the results support your reasoning in Question 5, part a.

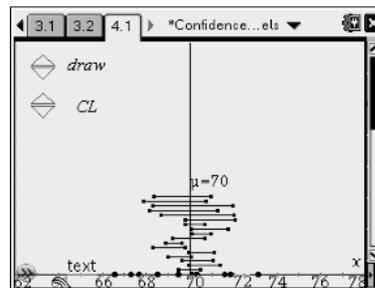
**Sample Answer:** The results should again suggest that about 45 out of 50 contain the population mean.

- c. The confidence *level* used to generate the confidence intervals on Page 3.2 was 90%. Based on your work in Questions 4 and 5, does 90% seem like a reasonable level? Why or why not?

**Sample Answer:** Students should recognize that 45 out of 50 is about 90%, which supports the claim that the confidence level for the work was 90%.

**Move to page 4.1.**

6. The up and down arrows (▲ and ▼) on page 4.1 control the confidence level. Note: the sample size stays constant. Set the confidence level to 90%. Generate 50 samples. Describe how well your confidence intervals support a confidence level of 90%.



**Sample Answer:** Student data should show that about 45 of the intervals contain the population mean, but a few students might have fewer or more captures.



7. a. If the confidence level is 95%, about what proportion of 50 confidence intervals generated from the same population for a given sample size do you think will contain the true population mean?

**Sample Answer:** Students should begin to see that about 95% of the 50, i.e. 47 or 48, will contain the true mean. Some might still be unsure.

- b. Use the down arrow (▼) to undo the confidence intervals you generated in Question 6. Set the confidence level to 95. Generate 50 confidence intervals. How do your results match your estimate in Question 7 part a?

**Sample Answer:** Most students will find from 46 to 49 of the intervals actually contain the population mean of 70.

**Teacher Tip:** Counting those that “touch” the vertical line might come up. The focus should be on the notion that for a given confidence level, you can find regularity in how many confidence intervals will include the actual population mean. If students need a “rule,” indicate that if it looks clearly as though it touches, count it in.

- c. Repeat Questions 6 parts a and b for another confidence level of your choice.

**Sample Answer:** Responses will vary depending on the confidence level they choose.

8. You can use the down arrow (▼) to return to a screen with no confidence intervals.
- a. Set the confidence level to 80%, and make a sketch of the resulting confidence interval, labeling the axis. Then set the confidence interval to 99% and make a sketch of that confidence interval using the same axis. What effect does changing the confidence level from a high level of confidence to a low level of confidence seem to have on the resulting intervals?

**Sample Answer:** For a given sample size, a high level of confidence will have a wider confidence interval than for a lower level of confidence. Students should have drawn two parallel confidence intervals with the 99% interval clearly wider than the 80% interval; for example, the 99% interval might go from 68.4 to 73.4, a span of 5, while the 80% interval might go from 69.5 to 72, a span of 2.5.

- b. What happens to the width of a confidence interval as the confidence level changes for the same sample size? Generate confidence intervals for three different confidence levels, sketch them, and label each with an appropriate confidence level.

**Sample Answer:** As the confidence level increases, the width of the confidence intervals increases. The sketches might show, for example, 80% CL: 68.5 to 71.5, a span of 3; 90% CL: 68.2 to 72, a span of 3.8; 99% CL: 67 to 73.4, a span of 6.4.



**Teacher Tip:** In general, as confidence level increases, the width of the corresponding confidence interval increases, all other factors being unchanged. One factor that also affects the width of confidence intervals is the standard deviation of the sampling distribution of the underlying statistic. In estimating the population mean when the population standard deviation is unknown, interval calculations are based on the standard deviation of the sample. Thus, two different samples of equal sample size (but different sample standard deviation) from a single population can give rise to two intervals at the same confidence level that have very different widths.

- c. Think about what a confidence level represents. How does this help justify your conclusion in Question 7 part b?

**Sample Answer:** In order to be more confident that the interval contains the population mean, a wider interval will give more of a chance that it will actually contain the mean.

**Teacher Tip:** Students might compare their graphs and answers with a classmate. Understanding this idea is important to understanding the whole notion of confidence level as a probability that the confidence interval produced by the process will contain the true mean.

9. An experiment led to the following: A 95% confidence interval for the mean number of hours that teenagers play video games is between 5.35 hours and 7.25 hours. Determine whether each statement is correct or incorrect.
- a. 95% of the time, the number of hours teenagers play video games is between 5.35 hours and 7.25 hours.

**Sample Answer:** This statement is incorrect; 95% confidence intervals do not contain 95% of all of the measurements.

- b. We are 95% confident that the mean number of hours that teenagers play video games is between 5.35 hours and 7.25 hours.

**Sample Answer:** This statement is correct.

- c. We are 95% confident that the mean number of hours that teenagers play video games is 6.3  $\pm$  0.95.

**Sample Answer:** This statement is correct; it is just a rephrasing of the interval using the center and distance to the ends of the interval.



- d. 95% of the confidence intervals we construct will give us the interval (5.35, 7.25).

**Sample Answer:** This statement is incorrect; confidence intervals usually give different results for each sample that is taken.

- e. There is a 95% probability that the mean number of hours that teenagers play video games is between 5.35 and 7.25 hours.

**Sample Answer:** This is incorrect; probability does not apply to an event that either has occurred or has not occurred. There is no chance involved in how many hours teenagers play video games; the chance is in the process of estimating that quantity from a sample.

10. Describe the difference between a confidence level and a confidence interval.

**Sample Answer:** A confidence level is a statement about the degree of certainty in the process of finding a confidence interval for an unknown parameter for a population. A confidence interval is the result of that process, is calculated based on a random sample from that population, and provides an interval of plausible estimates for the value of that parameter.

**TI-Nspire Navigator Opportunity: *Quick Poll***  
**See Note 5 at the end of the lesson.**

## Wrap Up

Upon completion of this lesson, teachers should ensure that students are able to understand:

- A confidence level applies to the *process* of generating a confidence interval.
- The process of generating a confidence interval for a specific confidence level will produce intervals that contain the true population parameter with relative frequency equal to that level of confidence.
- Changing the confidence level changes the width of the confidence interval; as the confidence level increases, the width of the confidence interval increases.



## TI-Nspire Navigator

### Note 1

#### Question 1, Name of Feature: Screen Capture

Use Screen Capture to look at the different samples students are generating and discuss what they think the true population mean is.

### Note 2

#### Question 2a, Name of Feature: Screen Capture

A Screen Capture will allow you to display the results to the whole class and to ask them what they might estimate for the population mean, given that some of intervals are “interesting” (i.e., they lie primarily to one side or the other of 70, or some are very short intervals and some are very long intervals).

### Note 3

#### Question 2d, Name of Feature: Screen Capture

A Screen Capture of each student’s Page 3.1 with 20 confidence intervals will show not only the variability possible but also the fact that in each case one or two of the intervals seem much different than the others. You might have them compare the proportions for the unlikely intervals and note that it seems to be 5% or 10% for each student’s set of confidence intervals.

### Note 4

#### Question 4, Name of Feature: Screen Capture

A Screen Capture will show how different sets of 50 confidence intervals seem to have a relatively constant proportion of those intervals that do contain the population mean of 70.

### Note 5

#### Question 10, Name of Feature: Quick Poll

Send questions to students either through a Quick Poll or a new .tns file to check their level of understanding. For example, you might use Question 9 as a true/false Quick Poll prior to discussing student responses. The discussion might focus on “Why might some of you think part d is incorrect? Does anyone want to explain why he or she thought it would be true?”